



Technical Report 80-1 August 1980

A TRANSFORMATION YIELDING AN ADDITIVE REPRESENTATION

OF DATA IN A TWO-WAY ARRAY

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This research was supported by the Advanced Research Projects Agency and monitored by the Office of Naval Research, Personnel and Training Research Programs, under Contract N00014-79-C-0214, NR 154-387. I would like to thank Michael Levine and Gerald Murch for helpful discussions while the report was in preparation.

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| REPORT DOCUMENTATION PAGE | | READ INSTRUCTIONS BEFORE COMPLETING FORM | |
|--|------------------------------------|--|--|
| 1. REPORT NUMBER | 2. GOVT ACCESSION NO. | 3. RECIPIENT'S CATALOG NUMBER | |
| Technical Report 80-1 | AD A 118666 | | |
| 4. TITLE (and Subtitle) | 5. TYPE OF REPORT & PERIOD COVERED | | |
| A Transformation Yielding An Addi | Tachnical manant | | |
| Representation Of Data In a Two-way | | Technical report | |
| Array | | 5. PERFORMING ORG. REPORT NUMBER | |
| 7. AUTHOR(9) | | 8. CONTRACT OR GRANT NUMBER(4) | |
| James A. Paulson | | _ | |
| | | N00014-79-C-0214 | |
| | | 12 PROCESUS SERVENT PROJECT TASK | |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS Psychology Department | | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61153N | |
| Portland State University | | RR 042-06-01 | |
| Portland, OR 97207 | | NR 154-429 | |
| 11. CONTROLLING OFFICE NAME AND ADDRESS | | 12. REPORT DATE | |
| Defense Advanced Research Projects Agency | | August 15, 1980 | |
| 1400 Wilson Blvd. | | 13. NUMBER OF PAGES | |
| Arlington, VA 22209 14. MONITORING AGENCY NAME & ADDRESS(II different from Controlling Office) | | 14 | |
| , | | 15. SECURITY CLASS. (of this report) | |
| Personnel and Training Research Programs | | Unclassified | |
| Office of Naval Research | | 15. DECLASSIFICATION/DOWNGRADING SCHEDULE | |
| Arlington, VA 22217 | | SCHEDULE | |
| 16. DISTRIBUTION STATEMENT (of this Report) | | <u> </u> | |
| Approved for public release; distribution unlimited. | | | |
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| 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) | | | |
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| 18. SUPPLEMENTARY NOTES | | | |
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DD 1 JAN 73 1473 EDITION OF 1 NOV 68 IS OBSOLETE S/N 0102-014-6601

A TRANSFORMATION YIELDING AN ADDITIVE REPRESENTATION OF DATA IN A TWO-WAY ARRAY

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It is common practice when seeking an additive representation of data in a two-way array to try out various transformations using Tukey's single degree of freedom for nonadditivity as an index of the extent of the deviation of the reexpressed data from additivity. It is shown that when data fit Tukey's model and an additive representation exists in the sense defined by Luce and Tukey (1964), the transformation required to obtain the additive representation is

$$f(y) = \log (\lambda y + 1 - \lambda \mu)$$
,

(1963).

where λ is the weight for the degree of freedom for nonadditivity. The transformation is unique up to a linear transformation. This result is in apparent conflict with power transformations suggested by Anscombe and Tukey



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Introduction

Suppose the structure of expected values, (μ_{ij}) , of a two-way array of random variables, (y_{ij}) , is given by Tukey's (1949) model with a single degree of freedom for nonadditivity,

$$\mu_{ij} = \mu + \alpha_i + \beta_j + \lambda \alpha_i \beta_j . \tag{1}$$

The following linear transformation yields a multiplicative representation of the array.

$$\lambda \mu_{\mathbf{i}\dot{\mathbf{j}}} - \lambda \mu + 1 = (1 + \lambda \alpha_{\dot{\mathbf{i}}})(1 + \lambda \beta_{\dot{\mathbf{j}}}) \ . \tag{2}$$

If $1+\lambda\alpha_i$ and $1+\lambda\beta_j$ are positive for all i, j, then we may take logarithms on both sides to obtain the additive representation

$$\log (\lambda \mu_{ij} - \lambda \mu + 1) = \log (1 + \lambda \alpha_i) + \log (1 + \lambda \beta_j) . \tag{3}$$

An additive representation of an array (μ_{ij}) exists in the sense defined by Luce and Tukey (1964) if there is a transformation of the μ_{ij} 's , $f(\mu_{ij})$, and functions of the row and column indices, r(i) and c(j) , such that

$$f(\mu_{ij}) = r(i) + c(j) \tag{4}$$

for all i, j and

$$f(\mu_{ij}) \ge f(\mu_{kl})$$

if and only if

for all i, j, k, l.

Luce and Tukey show that if such a transformation exists, then it is unique up to a linear transformation, as are the functions r(i) and c(j). Equation 3 shows that such a transformation exists for data described by Tukey's model, provided $l + \lambda \alpha_i$ and $l + \lambda \beta_j$ are positive for all i, j. If either $l + \lambda \alpha_i$ or $l + \lambda \beta_j$ are nonpositive for any i or j, then no additive representation exists. To see this, note that it follows immediately from Luce and Tukey's definition of an additive representation that the array (μ_{ij}) has such a representation only if the order of elements in any row is determined by the column function, c(j), and the order of elements in any column is determined by the row function, r(i). For example, for elements in row i it follows from Equation 4 that

$$\mu_{ij} - \mu_{ik} \stackrel{>}{=} 0$$

if and only if

$$f(\mu_{ij}) - f(\mu_{ik}) = c(j) - c(k) \ge 0$$
.

Equation 1 can be rewritten to show that the elements in row i are a linear function of $\beta_{\bf i}$, with slope 1 + $\lambda\alpha_{\bf i}$:

$$\mu_{ij} = \mu + \alpha_i + (1 + \lambda \alpha_i) \beta_j . \tag{5}$$

If $1+\lambda\alpha_i$ is negative, the order of the μ_{ij} 's in row i is the reverse of the order of the β_j 's. But in the additive representation given in Equation 3, $c(j) = \log (1+\lambda\beta_j)$, so the order of the c(j)'s is the same as the order of the β_j 's. Thus, the order of the μ_{ij} 's must differ from the order of c(j)'s, contradicting the possibility of an additive representation.

The transformation $\log (\lambda y - \lambda \mu + 1)$, just shown to be required to

obtain an additive representation if Tukey's model holds, has not been suggested before in the statistical literature, to the author's knowledge. Transformations of the form $\log (y + c)$ are mentioned in passing by Tukey (1949), Anscombe and Tukey (1963), and Box and Cox (1964), but the necessary connection between the parameters of Tukey's model and the value of c is not discussed. In fact, Anscombe and Tukey (1963) propose a different transformation based on the parameters of Tukey's model—the power transformation $y^{1-\lambda\mu}$. In the case $1{-}\lambda\mu$ = 0 , they suggest the transformation log y, which is the only case where their recommendation coincides with the transformation implied by the model. Kruskal (1965) and de Leeuw et al. (1976) have developed general numerical algorithms to find the monotone transformation that will render a given two-way array most additive, assuming only that such a transformation exists. It is beyond the scope of this paper to discuss the comparative virtues of the different approaches in detail, but there are a few points that should be mentioned. This might best be done in the context of in example.

Example

Figures la and lb are two slightly different ways of plotting data from an experiment of Carterette and Anderson (1979) on the scaling of loudness, using the bisection method. This method has the subject adjust a sound to the point where its loudness is halfway between the loudnesses of two given sounds. Carterette and Anderson propose a simple algebraic theory which implies that the subjective loudness of the sounds which the subjects give as their besection responses should be an additive function of the loudnesses of the sounds being bisected. A problem that arises in evaluating this theory empirically is that the subject's subjective sensation is an unknown function of the physical sound intensity expressed in decibels.

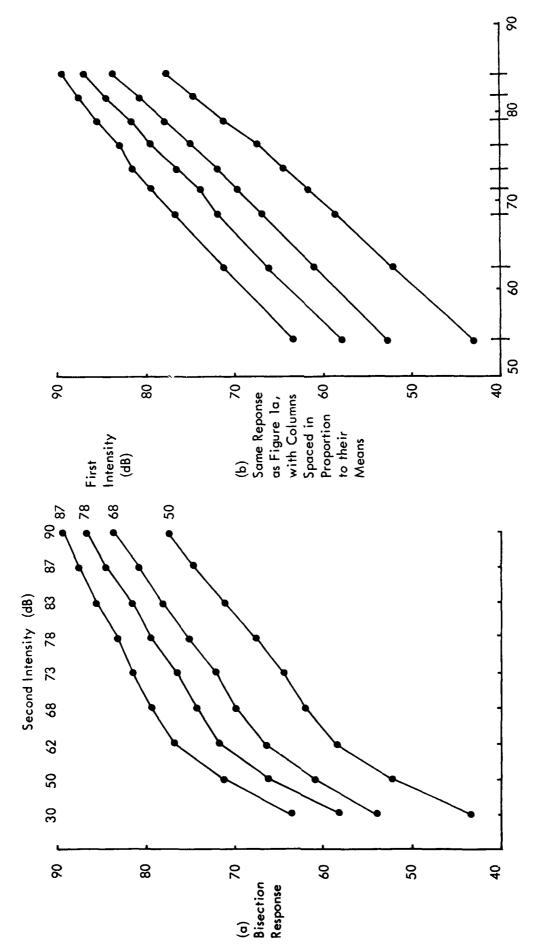
Carterette and Anderson circumvent this problem by applying the numerical algorithm of de Leeuw, et al. (1976) mentioned above to obtain a monotone transformation which renders their data as additive as possible. They then evaluate how well the transformed bisection response data fit the predictions of their theory. We shall use the data of the Carterette and Anderson to study the transformation produced by the de Leeuw et al. algorithm, the transformation implicit in Tukey's model, and the power transformation suggested by Anscombe and Tukey.

Figure la is a standard plot relating the raw bisection responses to the intensities of the first and second sounds in a pair to be bisected. Each point represents the mean of seventy observations made up of ten replications for each of seven subjects. The columns are equally spaced. Figure 1b presents the same data, except the columns are spaced according to their means. Spacing the columns according to their means provides a graphical method for determining whether or not Tukey's model fits the data. Equation 5 implies that the rows should plot as straight lines if Tukey's model fits. If the data are additive, the straight lines should be parallel. If not, they should meet at a common focal point. The intersection of the lines obtained from Equation 5 for any two rows is at $\beta_{\frac{1}{2}} = -1/\lambda$. The focal point of the lines representing the rows is therefore

$$y = \mu + \alpha_{i} + (1 + \lambda \alpha_{i})(-1/\lambda)$$

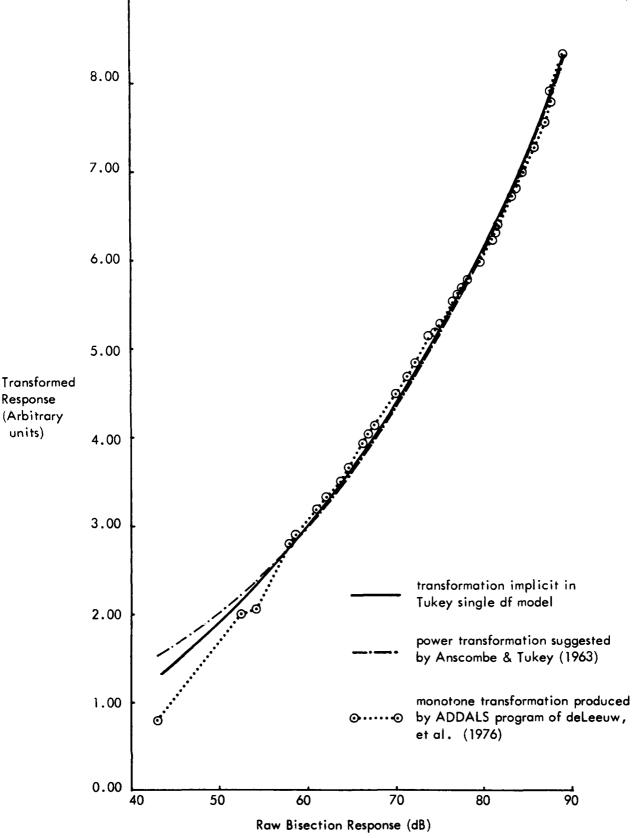
$$= \mu - 1/\lambda .$$
(5)

Note that the terms in the multiplicative representation given in Equation 2 can be interpreted as deviations from this focal value, as follows:



Raw loudness bisection responses, adapted from data of Carterette and Anderson (1979), plotted two different ways. Figures la, b.





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Figure 2. Transformations of raw bisection responses to attain additivity.

$$\lambda \mu_{ij} - \lambda \mu + 1 = \lambda(\mu_{ij} - \mu + 1/\lambda) ,$$

$$1 + \lambda \alpha_{i} = \lambda(\mu_{i} - \mu + 1/\lambda) ,$$

$$1 + \lambda \beta_{j} = \lambda(\mu_{j} - \mu + 1/\lambda) .$$
(6)

Furthermore, the reciprocal of λ is the overall mean of the deviations from the focal value. That is, Tukey's model essentially says that the expected deviation of an observation from the focal value is equal to the product of the mean deviation in that row, times the ratio of the mean column deviation to the overall mean deviation. (The role of rows and columns can obviously be switched.)

The data in Figure 1b fit Tukey's model fairly well. The lines are reasonably straight and seem like they might be converging to a focal point. For this data $\hat{\mu}=72.3$, $\hat{\lambda}=-.027$, and $\hat{\mu}-1/\hat{\lambda}=109.3$. The estimated focal value has a substantive interpretation. According to Licklider (1951), 110-115 dB is about the point where sound begins to be too intense for comfort, for a person who has not adapted to listening to loud sounds. Sounds this loud give rise to nonauditory sensations such as tickle and pain.

Figure 2 shows the effect of three transformations on the raw bisection response data: the ADDALS transformation of de Leeuw, et al. (1976) employed by Carterette and Anderson, the power transformation suggested by Anscombe and Tukey (1963), and the transformation proposed in this paper. The latter two transformations were further transformed by linear regression to obtain comparability with the ADDALS transformation values. This is legitimate because linear transformation has no effect on additivity. The zero and unit of the ADDALS transformation, being arbitrary, were selected by Carterette and Anderson for convenience.

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Carterette and Anderson conclude that, on the whole, the ADDALS transformation succeeds in rendering the data additive, though they note some slight but statistically significant nonadditivity remaining in individual analyses of some subjects. Figure 2 suggests that they would have obtained similar results with the other two transformations, which are very close to the ADDALS values except for the lowest three points. The correlations between ADDALS values and the other two transformations are .997 for the transformation implied by Tukey's model and .995 for the power transformation suggested by Anscombe and Tukey (1963). While these high correlations suggest that the results would come out the same, no matter which of the three transformation was employed, it would be wrong to conclude that there are no meaningful differences between the transformations. For reasons to be described presently, the modified log transformation and the power transformation do yield almost literally identical results with data falling in the relatively restricted range of the present data. However, even though these two transformations correlate very highly with the ADDALS transformation, they differ from it systematically, tending to exceed the ADDALS values at the extremes while undershooting them in the middle range. Anderson and Shanteau (1977) emphasize the inadequacy of correlation as a test of agreement between theory and prediction. These data illustrate their point again.

The general monotone transformation differs most markedly from the other two with respect to smoothness. While the overall increasing trend is steady, it is interspersed with minor fits and starts. For example, note the little steps in the curve for values of the transformed response just above 2.00, 4.00, and 5.10, and the little jump that occurs at about 4.80.

As noted above, when the ratio of the largest data point to the smallest is not too big, the transformations $y^{1-\lambda\mu}$ and log $(\lambda y - \lambda\mu + 1)$ produce very

similar results with regard to additivity. Tukey (1957) defines a measure of the strength of a transformation which helps to explain why this is so. If z is the transformation, Tukey's measure of strength, as one goes from y_1 to y_2 in the original metric, is

$$S(y_1, y_2) = \frac{\left(\log \frac{dz}{dy}\right)_{y=y_1} - \left(\log \frac{dz}{dy}\right)_{y=y_2}}{\log y_2 - \log y_1} . \tag{7}$$

The rationale for this measure is a bit subtle; see Tukey (1957) for details. For power transformations y^p the strength is 1-p, independent of the choice of y_1 and y_2 . Also, the strength of the transformation log y is 1, independent of y_1 and y_2 . Hence, the sequence of transformations

...,
$$y$$
, $y^{\frac{1}{2}}$, $\log y$, $y^{-\frac{1}{2}}$, y^{-1} , ...

is equally spaced in terms of this measure. The strength of the more general transformation

$$z = (y + c)^{p}, p \neq 0,$$

 $log(y + c), p = 0,$

depends on y_1 and y_2 . However, power series expansions of $\log \frac{1+x}{1-x}$ applied to Equation 7 show that a good first approximation to the strength is given by

$$S = (1 - p) \frac{y}{y + c}$$
 (8)

where y is the midpoint of y and y . Applying this approximation to log (y - μ + $1/\lambda)$ at y = μ yields

$$S = (1 - 0) \frac{\mu}{\mu + (1/\lambda - \mu)}$$
$$= \lambda \mu ,$$

which is the same as the strength of the transformation $y^{1-\lambda\mu}$. Thus, it is not surprising that $\log{(y-\mu+1/\lambda)}$ and $y^{1-\lambda\mu}$ yield similar results close to the mean of the original data. If the range of the data is not great, one never gets far enough from the mean for the difference in strength to become noteworthy.

Discussion

Researchers sometimes object to the use of transformations because they distort comparative differences between rows as one goes from one column to another. There are two ways one might respond to this objection. One is to note that the original metric chosen to represent the data may be inappropriate, and to insist on its use may cause the investigator to miss significant regularities in his or her data. In such situations, a simple power transformation of the original data, such as the reciprocal or square root, may be as meaningful as the original metric, while leading to more interpretable patterns of results.

A second reply might be that even if the original metric is quite appropriate, the usual statistical analyses applied to the data expressed in this metric might lead to erroneous conclusions. This is particularly so if one detects significant nonadditivity in the original data array, concludes that the differences produced by one factor are inconsistent over levels of the other factor, and leaves it at that. Additivity refers to the invariance of differences between row elements as one goes from column to column in the array. Other invariant relationships might hold, though additivity does not, and seeking a transformation to attain additivity might lead one to notice them. For example, when Tukey's model holds, ratios of intervals between rows are invariant from column to column. Consider any

four rows, and let their row indices be i = 1, 2, 3, 4. Then it follows from Equation 1 that

$$\frac{\mu_{1j} - \mu_{2j}}{\mu_{3j} - \mu_{4j}} = \frac{(\alpha_1 - \alpha_2)^{\lambda \beta_j}}{(\alpha_3 - \alpha_4)^{\lambda \beta_j}}$$

$$= \frac{\alpha_1 - \alpha_2}{\alpha_3 - \alpha_4},$$
(9)

independent of j. It was noted earlier that appropriate graphical representation of the original data can be useful in determining if Tukey's model holds. If it appears that it holds for a given set of data, it makes sense to apply the transformation implied by the model. Then a more rigorous test of the fit of the model can be carried out by testing the transformed data for additivity.

The force of these replies depends on the simplicity, or nonarbitrariness, of the transformation used, which in turn depends on the details of the situation. Thus, when the transformation proposed in this paper or a simple power transformation will work, they would seem to be preferable to the general monotone transformations. It was noted above that the latter will usually have a lack of smoothness which makes them appear somewhat arbitrary. However, one should not hesitate to employ the more complicated algorithms in situations where the simple approaches fail.

In choosing between the power transformation and the modified logarithmic transformation proposed in this paper one might be guided by substantive considerations. The exponent of the implied power transformation, or the focal point of the modified logarithmic transformation may make theoretical sense. If not, the choice would seem to be a matter of taste. In that case, the author prefers the transformation implicit in the model used to assess the nonadditivity.

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